A new equilibrium simulation procedure with discrete choice models

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ABSTRACT: Many microeconometric models of discrete choice include alternative-specific constants meant to account for (possibly besides other factors) the density or accessibility of particular types of alternatives. A notable area of application is labour supply, where for example part-time jobs vs. full-time jobs might be more or less accessible. The most common use of these models is the simulation of tax-transfer reforms. The simulation is usually interpreted as a comparative statics exercise, i.e. the comparison of different equilibria induced by different policy regimes. The simulation procedure, however, typically keeps fixed the estimated alternative-specific constants. In this note we argue that this procedure is not consistent with the comparative statics interpretation. Since the constants reflect the number of jobs and since the number of people willing to work changes as a response to the change in tax-transfer regime, the new equilibrium induced by the reform implies that the constants should also change. A structural interpretation of the alternative-specific constants leads to the development of a simulation procedure consistent with the comparative statics interpretation. The procedure is illustrated with a simulation of alternative reforms of the income support policies in Italy.


1. INTRODUCTION

A common practice in the specification of models of labour supply based on the discrete choice approach consists of introducing alternative-specific constants, which should account for a number of factors such as the different density or accessibility of different types of jobs, search or fixed costs and systematic utility components otherwise not accounted for\(^1\). When using the models for evaluating reforms of institutions or policies with long-run effects (such as tax-benefit reforms), comparative statics is the appropriate perspective: i.e. we want to compare two different equilibria induced by two different policies. With the notion of equilibrium we refer in general to a scenario in which the economic agents make optimal choices (i.e. they choose the best alternative among those available in the opportunity set) and their choices are mutually consistent or feasible. In what follows, we will make the specific assumption that mutual consistency of the agents’s choices implies that the number of available jobs be equal to the number of workers willing to be matched to those jobs. To the extent that the alternative-specific constants reflect also the demand side (e.g. the availability of jobs), a new equilibrium induced by a reform should entail a change of the alternative-specific constants. Instead the standard simulation procedure leaves those constants unchanged: in this paper we argue that the standard procedure is not consistent with the comparative statics interpretation of the simulation results. Based on a structural interpretation of the alternative-specific constants, we propose a simulation procedure that is consistent with comparative statics.

In the basic discrete choice framework, the household chooses among \(H+1\) alternatives or “job” types \(j = 0, 1, \ldots, H\), with \(j = 0\) denoting non-participation (a “non-market job”). Let \(V(i, j; w, T) + \varepsilon_j\) denote the utility attained by household \(i\) if a job of type \(j\) is chosen, given wage rate \(w\) and tax-benefit regime \(T\), where \(V(i, j; w, T)\) is the systematic part (containing observed variables) of the utility function and \(\varepsilon_j\) is a random component. Depending on the application and the available data, the job types might be defined in terms of one or many of the following attributes: weekly hours of work, sector of employment, occupational level, type of contract etc. By assuming that \(\varepsilon_j\) is i.i.d. Type I extreme value, we get the Conditional Logit expression for the probability that a job of type \(j\) is chosen by household \(i\):

\[
P(i, j; w, T) = \frac{\exp\left\{V(i, j; w, T)\right\}}{\sum_{b=0}^{H} \exp\left\{V(i, b; w, T)\right\}}
\]  

(1)
Model (1) usually does not fit the data very well. For example Van Soest (1995) notes that the model over-predicts the number of people working part-time. More generally, certain types of jobs might differ according to a number of systematic factors that are not accounted for by the observed variables contained in \( V(\cdot) \): (a) availability or density of job-types; (b) fixed costs; (c) search costs; (d) systematic utility components. What might be called the “dummies refinement” is a simple way to account for those factors. Let us define subsets \( \{ S_{\epsilon} \} \) of the set of job types \( 0, 1, \ldots, H \) and the corresponding indicator functions \( \{ I(j \in S_{\epsilon}) \} \) such that \( I(\epsilon) = 1 \) if and only if \( \epsilon \) is true. Clearly the definition of the subsets should reflect some hypothesis upon the differences among the job types with respect to the factors (a) – (d) mentioned above. Now we specify the choice probability as follows

\[
P(i, j; w, T) = \frac{\exp \left( V(i, j; w, T) + \sum_{\epsilon} \gamma_{I(j \in S_{\epsilon})} \right)}{\sum_{\delta=0}^{H} \exp \left( V(i, \delta; w, T) + \sum_{\epsilon} \gamma_{I(k \in S_{\epsilon})} \right)}
\]

Expression (2) can be interpreted as embodying the assumption that certain jobs, beyond the contributions attributable to the observed characteristics, bring a systematic additive utility contribution, due to a number of unobserved systematic factors including their accessibility. More generally, the systematic unobserved contributions could be entered in a non-additive forms or could be measured in terms of income rather than utility. For example, another common procedure consists of subtracting from the income term (in the utility function) a parameter (usually called “participation cost” or “fixed cost of working”) whenever the job is a “market job”. In what follows, we will refer to the formulation of expression (2).

The main use of microeconometric models of labour supply consists of the simulation of tax-benefit reforms. The standard simulation proceeds as follows. Once \( V(\cdot) \) and the \( \{ \gamma_{\epsilon} \} \) are estimated, the current tax-benefit regime \( T \) is replaced by a “reform” \( R \) and a new distribution of choices is simulated using expression (2). All the authors adopting the “dummies refinement” so far have performed the simulations by leaving the \( \{ \gamma_{\epsilon} \} \) unchanged. The policy simulation is most commonly interpreted as a comparative statics exercise, where different equilibria – induced by
different tax-transfer regimes – are compared. In this paper we claim that the standard procedure in general is not consistent with the comparative statics interpretation. In the following sections we adopt a notion of equilibrium that requires the number of available market jobs to be equal to the number of people matched to those jobs. Then, since the $\{\gamma_t\}$ reflect – at least in part, depending on the interpretations – the number and the composition of available market jobs, and since the number of people willing to work and their distribution across different job types in general change as a consequence of the reforms, it follows that in general the $\{\gamma_t\}$ must also change. Moreover, if it is assumed that the mechanism leading to the equilibrium is a change of the wage rates, then the wage rates should change as well. Sometimes the standard simulation procedure is interpreted as a long-term scenario with a perfectly elastic labour demand and therefore constant wage rates: even in this scenario, however, the $\{\gamma_t\}$ must change since in general the number of employed people and jobs do change.

In what follows we present a structural interpretation of the “dummies refinement” that leads very naturally to a simulation procedure consistent with comparative statics.$^3$

The procedure is explained in Sections 2, 3 and 4. Section 5 illustrates an empirical example. Section 6 contains the conclusions.

2. A STRUCTURAL INTERPRETATION OF THE “DUMMIES REFINEMENT”

We consider here a single individual. The generalization to couples is developed in Section 4. Building on Dagsvik (1994), a series of papers$^4$ adopt an approach where there are “many” jobs that belong to each type $j$ and a particular job $\zeta$ of type $j$ produces a utility level $V(i,j;w_j,T)+\varepsilon_j(\zeta)$, so that $V(i,j;w_j,T)+\varepsilon_y$ is to be interpreted as follows:

$$V(i,j;w_j,T)+\varepsilon_y = \max_{\zeta} V(i,j;w_j,T)+\varepsilon_{\zeta}(\zeta).$$

(3)

We let $g_j$ denote the number of available jobs of type $j$. Type 0 refers to non-market jobs, or activities outside the labour market: therefore $g_0$ denotes the number of non market opportunities. The terms $g_j$ can be interpreted as reflecting the demand side. In general it might be both job-specific and individual-specific but for simplicity of exposition we treat it here as common to all individuals.

By assuming that $\varepsilon_y$ is i.i.d. Type I extreme value, the probability that individual $i$ chooses a job
of type $j$ turns out to be (e.g. Dagsvik 1994; Aaberge et al. 1995, 1999; Dagsvik 2000):

$$M(i, j; w_i, g, T) = \frac{\exp\{V(i, j; w_i, T)\} g_j}{\sum_{j=0}^H \exp\{V(i, h; w_i, T)\} g_h}$$

(4)

where $g = (g_0, g_1, ..., g_H)'$.

Dagsvik (2000) shows that expressions (4) can be derived as a special case of a model where the agents (firms and workers) play a game leading to stable matching equilibrium, e.g. the deferred acceptance game (Roth 2008). By adopting his approach and with sufficiently rich data on job characteristics, vacancies, individual productivities and job search activities, it might be possible to model and empirically identify the labour market equilibrium as a stable matching. However, in this exercise we follow a different route. Expression (4) is also compatible with alternative representations of the equilibrium process. First, we assume that any individual can in principle choose among all the jobs available on the market: the number of jobs of type $j$ available to individual $i$ is the same for everyone and is also equal to the total number of jobs of type $j$ available on the market. Therefore $J_j$, the number of jobs available to individual $i$, is also the total number of available jobs. We might allow for different opportunity sets available to different types of individuals: for example in Section 4.2 and in the empirical illustration of Section 5 we will assume gender-specific individual opportunity sets. However it remains true in our assumptions that the opportunity set available to any individual of a given type coincide with total opportunity set available to that type. This is clearly a special assumption. In the more general case of strictly individual-specific opportunity sets (as in Dagsvik 2000) there is no straightforward relationship between the number of jobs available to any particular individual and the total number of available jobs. Second, we adopt a simple, “textbook”, concept of equilibrium. The equilibrium wage distribution (or equivalently, the equilibrium moment vector of the wage distribution) is such that the number of available market jobs of type $j$ and the (expected) number of people who choose a job of type $j$ are equal. The equilibrium means that there is a job of type $j$ for everyone choosing a job of type $j$. The model does not explicitly represent the process by which individual $i$ is allocated to a particular job among the jobs of type $j$: the random component $\epsilon_{ij}$ (which accounts for the specific utility contribution of unobserved characteristics of individual $i$ and/or jobs of type $j$) determines which specific job (among jobs of type $j$) is allocated to individual $i$.

In empirical applications we are typically interested in distinguishing among particular job-types, such part-time vs. full-time or temporary contracts vs. permanent contracts etc. Let us define
$S_1, ..., S_L$ as $L$ mutually exclusive sets of job-types. We assume the sets are not exhaustive, so that the not included jobs represent a reference category.

We then define

$$J = \sum_{b>0} g_b = \text{total number of market jobs.} \quad (5)$$

Next we specify, for $b > 0$:

$$g_b = \left\{ \begin{array}{ll}
\alpha \exp(\gamma_1) & \text{if } b \in S_1 \\
... & \\
\alpha \exp(\gamma_L) & \text{if } b \in S_L \\
\alpha & \text{otherwise}
\end{array} \right. \quad (6)$$

where $\alpha$ is a positive constant and $\gamma_1, ..., \gamma_L$ are parameters to be estimated. Expression (6) assumes that the distribution of jobs is uniform with “peaks” corresponding to the sets of job-types $S_1, ..., S_L$. By replacing $g_j$ with $\left( \frac{g_j}{J g_0} \right)$ at the numerator and $g_b$ with $\left( \frac{g_b}{J g_0} \right)$ at the denominator of expression (4) and using expression (6), we arrive at a “dummy – refinement” specification:

$$M(i, j; w_i, T) = \frac{\exp\left\{ V(i, j; w_i, T) + \gamma_0 I \left( j > 0 \right) + \sum_{\ell=1}^{L} \gamma_\ell I \left( j \in S_\ell \right) \right\}}{\sum_{b=0}^{L} \exp\left\{ V(i, b; w_i, T) + \gamma_0 I \left( b > 0 \right) + \sum_{\ell=1}^{L} \gamma_\ell I \left( b \in S_\ell \right) \right\}}. \quad (7)$$

The dummies’ coefficients $\gamma$ have the following interpretation:

$$\gamma_0 = \ln \left( \frac{\alpha J}{g_0} \right) \quad (8)$$

$$\gamma_\ell = \ln \left( \frac{J_\ell \frac{1}{n_\ell}}{\alpha J/ n_\ell} \right) \quad (9)$$

where

$$J_\ell = \sum_{k \in S_\ell} g_k = \text{number of market jobs of type } \in S_\ell$$

and

$$n_\ell = \text{number of types in } S_\ell, \ell = 1, ..., L.$$
Other factors besides the jobs opportunity density (such as unobserved systematic costs or benefits specific of different job types) are not incompatible with expressions (8) and (9): more generally $g_0$ and $\alpha$ might be interpreted as normalizing constants that include the effect of those other factors.

Note that given the notion of equilibrium defined above, the total number of available jobs of a given type is equal to the total number of workers holding a job of that type. Therefore $g_0$ and $\alpha$ can be retrieved using expressions (8) - (9) together with the estimated values of the coefficients $\gamma_0, \ldots, \gamma_L$ and the observed values of $J, J_\ell$ and $n_\ell, \ell = 1, \ldots, L$.

3. THE EQUILIBRIUM SIMULATION PROCEDURE

In this section, for simplicity of exposition, we consider the case where $\gamma_1 = \ldots = \gamma_L = 0$, so that the model contains only one dummy:

$$M(i, j; w_i, \gamma_0, T) = \frac{\exp\left\{V(i, j; w_i, T) + \gamma_0 I(j > 0)\right\}}{\sum_{k=0}^{L} \exp\left\{V(i, k; w_i, T) + \gamma_0 I(k > 0)\right\}}. \quad (10)$$

The general case is treated in Section 4. Let $\mu$ denote the moments characterizing the wage distribution. For example, if it is assumed that the wage rates belong to a log-normal distribution, the vector $\mu$ will contain the mean and the variance. Accordingly, $w_i(\mu)$ is the wage rate of individual $i$ given the wage distribution characterized by $\mu$. For simplicity of exposition we assume here that the individual wages belong to a common wage distribution. More generally we could assume that wage rates are drawn from job- and/or individual-specific distributions: in the empirical example of Section 5 we introduced two gender-specific wage distributions. We assume that the number of available jobs $J$ depends on the wage distribution, i.e. on the moments $\mu$:

$$J = J(\mu). \quad (11)$$

Since $\gamma_0$ depends on $J$ (according to expression (8)) and $J$ depends on $\mu$ (according to expression (11)), we write

$$\gamma_0 = \gamma_0(\mu) \quad (12)$$

Therefore the probability that individual $i$ chooses a job of type $j$, given the wage distribution moments $\mu$ and the tax-benefit regime $T$ is

$$M(i, j; w_i(\mu), \gamma_0(\mu), T) = \frac{\exp\left\{V(i, j; w_i(\mu), T) + \gamma_0(\mu) I(j > 0)\right\}}{\sum_{k=0}^{L} \exp\left\{V(i, k; w_i(\mu), T) + \gamma_0(\mu) I(k > 0)\right\}}. \quad (13)$$
With $w_i(\mu_T)$ we denote the wage rate of household $i$ in the equilibrium wage distribution induced by tax-benefit regime $T$.

It is important to distinguish the case of elastic labour demand from the limit cases of perfectly inelastic and perfectly elastic labour demand.

### 3.1. Elastic demand

Assuming that the observed (or simulated) choices under the current tax-benefit regime $T$ correspond to an equilibrium, we must have:

$$
\sum_i \sum_{j>0} M(i, j; w_i(\mu_T), \gamma_0(\mu_T), T) = J(\mu_T)
$$

i.e. the wage distribution moments must be such that the (expected) number of people choosing to be employed (the term on the left-hand side) is equal to the number of available jobs (the term on the right-hand side). In a comparative statics perspective, an analogous condition must hold under the reformed tax-benefit regime $R$:

$$
\sum_i \sum_{j>0} M(i, j; w_i(\mu_R), \gamma_0(\mu_R), R) = J(\mu_R)
$$

where $\mu_R$ denotes the moment vector of the new equilibrium wage distribution.

### 3.2. Perfectly elastic demand

When the demand for labour is perfectly elastic, the market is always in equilibrium at the initial wage rate. However, since the number of working people in general will change under a new tax-benefit rule and since the number of jobs in equilibrium must be equal to the number of people willing to be matched to those jobs, it follows that the parameter

$$
\gamma_0 = \ln\left(\frac{\alpha_j}{g_0}\right)
$$

must change. Let us rewrite expression (8) as $J = \left(g_0/\alpha\right)e^{\gamma_0}$. Then the equilibrium condition can be written as follows:

$$
\sum_i \sum_{j>0} M(i, j; w_i(\mu_T), \gamma_0(\mu_T), T) = \left(g_0/\alpha\right)e^{\gamma_0}.
$$
In this case the moments $\mu_i$ remain unchanged. Instead $\gamma_0$ must be directly adjusted (to the equilibrium value $\gamma_{0R}$ so as to fulfil condition (16). The case with a fixed wage distribution and the labour demand absorbing any change in labour supply actually corresponds to the scenario implicitly assumed in most tax-benefit simulations: however those simulations do not take condition (16) into account.

There is a special case of the perfectly elastic labour demand scenario where the standard simulation procedure might be considered as appropriate. So far we have assumed that $g_0$ (the number of non-market opportunities) is fixed. It might be argued that as $J$ changes also $g_0$ might change, e.g. because market jobs provide goods and services that are complements or substitutes to non-market activities. Let us consider again the one-dummy model. If we make the very special assumptions that $g_0$ varies in the same proportion as $J$ and that labour demand is perfectly elastic, then we have a scenario where both the wage rate and $\gamma_0$ remain constant, thus providing an equilibrium interpretation of the standard simulation procedure.

### 3.3. Perfectly inelastic demand

In the special case of a perfectly inelastic demand (zero elasticity), the number of jobs remains fixed but the wage rate must be adjusted so that the number of people willing to work under the new regime is equal to the pre-reform number of jobs:

$$\sum_i \sum_{j=0} M(i,j;w_i(\mu_k),\gamma_0(\mu_\gamma),R) = J(\mu_\gamma)$$

### 3.4. Implementation of the equilibrium simulation procedure

The implementation of the equilibrium simulation procedure requires to specify how $J$ depends on $\mu$. In the empirical example of Section 5, for illustrative purposes we will adopt the simple assumption that $J$ depends on the mean of the wage distribution according to a constant-elasticity relationship such as $J = K\mu^{-\eta}$, where $\mu$ denotes the mean of the wage rate distribution, $-\eta$ is the elasticity of labour demand and $K$ is a constant. Individual wage rates are shifted together with the mean $\mu$ and maintain the same rank position in the distribution. However, the procedure is completely general: in Section 4 we illustrate various extensions where it make sense to allow for individual- or job-specific wage distributions, cross-elasticities of labour demand for different types of individuals etc.
4. EXTENSIONS

The basic framework illustrated in Section 3 can be extended in many directions. Hereafter we consider the extension to the not uniform density of market jobs and the extension to couples. In this section we go back to the original formulation of Section 2 where $\gamma_0, \ldots, \gamma_L$ in general can differ from zero, i.e. the model contains $L+1$ dummies.

4.1. Non uniform density of market jobs

As in expression (7), we might want to specify a non-uniform conditional density for the market jobs. Let us consider again a single person. In this case we write $f = f(\mu)$ and $f_\ell = f_i(\mu), \ell = 1, \ldots, L$, which implies the relationships $\gamma_0 = \gamma_0(\mu)$ and $\gamma_\ell = \gamma_\ell(\mu)$.

The probability that individual $i$ is matched to a job of type $j$ is

$$M(i, j; w_i(\mu_i), \gamma(\mu_i), T) = \frac{\exp\left\{V(i, j; w_i(\mu_i), T) + \gamma_j(\mu_i)I(j > 1) + \sum_{\ell=1}^{L} \gamma_\ell(\mu_i)I(j \in S_\ell)\right\}}{\sum_{j=0}^{L} \exp\left\{V(i, j; w_i(\mu_i), T) + \gamma_j(\mu_i)I(j > 1) + \sum_{\ell=1}^{L} \gamma_\ell(\mu_i)I(j \in S_\ell)\right\}}, \quad (18)$$

where we have defined $\gamma(\mu_i) = (\gamma_0(\mu_i), \gamma_1(\mu_i), \ldots, \gamma_L(\mu_i))^T$.

The probability that individual $i$ is matched to a market job is $\sum_{j=0}^{L} M(i, j; w_i(\mu_i), \gamma(\mu_i), T)$, while

$$\sum_{j=0}^{L} M(i, j; w_i(\mu_i), \gamma(\mu_i), T)$$

is the probability that individual $i$ is matched to a market job of type $j \in S_\ell$. By solving expression (8) for $f = (g_0/\alpha)e^{\gamma_1}$ and expression (9) for $f_\ell = g_0e^{\gamma_0+\gamma_\ell}$, we find that the equilibrium conditions for a reform $R$ are respectively:

$$\sum_i \sum_{j=0}^{L} M(i, j; w_i(\mu_i), \gamma(\mu_i), R) = (g_0/\alpha)e^{\gamma_1(\mu_i)}$$

$$\sum_i \sum_{j=0}^{L} M(i, j; w_i(\mu_i), \gamma(\mu_i), R) = g_0e^{\gamma_0(\mu_i)+\gamma_\ell(\mu_i)}, \ell = 1, \ldots, L, \quad (19)$$

with elastic demand;
\[ \sum_{i}^{H} \sum_{j=1}^{J} M(i, j; w_j(\mu_j), \gamma_R, R) = (g_0/\alpha) e^{\gamma_R} \]  
(20)

\[ \sum_{i}^{H} \sum_{j=1}^{J} M(i, j; w_j(\mu_j), \gamma_R, R) = g_0 e^{\gamma_R}, \ell = 1, \ldots, L \]

with perfectly elastic demand and

\[ \sum_{i}^{H} \sum_{j=0}^{J} M(i, j; w_j(\mu_R), \gamma(\mu_i), R) = (g_0/\alpha) e^{\gamma(\mu_i)} \]  
(21)

\[ \sum_{i}^{H} \sum_{j=1}^{J} M(i, j; w_j(\mu_R), \gamma(\mu_i), R) = g_0 e^{\gamma(\mu_i)}, \ell = 1, \ldots, L \]

with perfectly inelastic demand.

4.2. Couples

When analyzing the simultaneous labour supply decisions of couples we might want to distinguish the opportunity set available to males (M) and females (F). The previous notation and the matching probabilities are generalized accordingly:

\[ M(i, j, j_M; w_i, w_j, T) = \frac{e^{V(i, j, j_M, w_i, w_j, T)} + \sum_{x=M, F} \{ \gamma_{x, i} I(j_x > 0) + \sum_{j_x = 1}^{J_x} \gamma_{x, j_x} I(j_x < S_{x}) \}}{\sum_{x=M, F} \sum_{j_x = 0}^{J_x} \exp \left( V(i, j, j_M, w_i, w_j, T) + \sum_{x=M, F} \{ \gamma_{x, i} I(j_x > 0) + \sum_{j_x = 1}^{J_x} \gamma_{x, j_x} I(j_x < S_{x}) \} \right)} \]  
(22)

where \( w_i = (w_j, w_M) \) and \( \gamma = (\gamma_F, \gamma_M)^\top \). For \( \alpha = F \) or M, expressions (8) and (9) are generalized as follows:

\[ \gamma_{0x} = \ln \left( \frac{\alpha f_x}{g_{0x}} \right), \gamma_{lx} = \ln \left( \frac{f_x}{\alpha f_x} \right), \ell = 1, \ldots, L_{x}. \]  
(23)

We then specify the gender-specific labour demand functions:

\[ J_x = J_x(\mu) \]  
(24)

\[ J_{f_x} = J_{f_x}(\mu), \ell = 1, \ldots, L_{f_x} \]  
(25)

where \( \mu \) now denotes the moments of the joint distribution of the partners’ wage rates.
Expressions (23), (24) and (25) imply a mapping such as:

\[ \gamma_{0x} = \gamma_{0x}(\mu), \]
\[ \gamma_{\ell x} = \gamma_{\ell x}(\mu), \ell = 1, \ldots, L_{x}. \]  

(26)

We define the equilibrium wage distribution as the distribution that – for each gender – equates the number of available market jobs to the number of individual matched to a market job, therefore the post-reform equilibrium conditions are

\[ \sum_{i} \sum_{j_{F} > 0} \sum_{j_{M} = 0}^{H_{M}} M(i, j_{F}, j_{M}; w_{i}(\mu_{R}), \gamma(\mu_{R}), R) = (g_{0F}/\alpha) e^{\gamma_{0x}(\mu_{F})}, \]
\[ \sum_{i} \sum_{j_{F} \in S_{F}} \sum_{j_{M} = 0}^{H_{M}} M(i, j_{F}, j_{M}; w_{i}(\mu_{R}), \gamma(\mu_{R}), R) = g_{0F} n_{iF} e^{\gamma_{0x}(\mu_{F}) + \gamma_{EF}(\mu_{R})}, \ell = 1, \ldots, L, \]
\[ \sum_{i} \sum_{j_{M} > 0} \sum_{j_{F} = 0}^{H_{F}} M(i, j_{F}, j_{M}; w_{i}(\mu_{R}), \gamma(\mu_{R}), R) = (g_{0M}/\alpha) e^{\gamma_{0M}(\mu_{M})} \]
\[ \sum_{i} \sum_{j_{M} \in S_{M}} \sum_{j_{F} = 0}^{H_{F}} M(i, j_{F}, j_{M}; w_{i}(\mu_{R}), \gamma(\mu_{R}), R) = g_{0M} n_{iM} e^{\gamma_{0M}(\mu_{M}) + \gamma_{MR}(\mu_{R})}, \ell = 1, \ldots, L. \]

(27)

for the case with elastic demand;

\[ \sum_{i} \sum_{j_{F} > 0} \sum_{j_{M} = 0}^{H_{M}} M(i, j_{F}, j_{M}; w_{i}(\mu_{F}), \gamma_{R}, R) = (g_{0F}/\alpha) e^{\gamma_{0R}} \]
\[ \sum_{i} \sum_{j_{F} \in S_{F}} \sum_{j_{M} = 0}^{H_{M}} M(i, j_{F}, j_{M}; w_{i}(\mu_{F}), \gamma_{R}, R) = g_{0F} n_{iF} e^{\gamma_{0R}(\mu_{F}) + \gamma_{FR}(\mu_{R})}, \ell = 1, \ldots, L, \]
\[ \sum_{i} \sum_{j_{M} > 0} \sum_{j_{F} = 0}^{H_{F}} M(i, j_{F}, j_{M}; w_{i}(\mu_{F}), \gamma_{R}, R) = (g_{0M}/\alpha) e^{\gamma_{0M}(\mu_{M})} \]
\[ \sum_{i} \sum_{j_{M} \in S_{M}} \sum_{j_{F} = 0}^{H_{F}} M(i, j_{F}, j_{M}; w_{i}(\mu_{F}), \gamma_{R}, R) = g_{0M} n_{iM} e^{\gamma_{0M}(\mu_{M}) + \gamma_{MR}(\mu_{R})}, \ell = 1, \ldots, L. \]

(28)

for the case with perfectly elastic demand and

\[ \sum_{i} \sum_{j_{F} > 0} \sum_{j_{M} = 0}^{H_{M}} M(i, j_{F}, j_{M}; w_{i}(\mu_{R}), \gamma_{R}, R) = (g_{0F}/\alpha) e^{\gamma_{0R}(\mu_{F})} \]
\[ \sum_{i} \sum_{j_{F} \in S_{F}} \sum_{j_{M} = 0}^{H_{M}} M(i, j_{F}, j_{M}; w_{i}(\mu_{R}), \gamma_{R}, R) = g_{0F} n_{iF} e^{\gamma_{0R}(\mu_{F}) + \gamma_{FR}(\mu_{R})}, \ell = 1, \ldots, L, \]
\[ \sum_{i} \sum_{j_{M} > 0} \sum_{j_{F} = 0}^{H_{F}} M(i, j_{F}, j_{M}; w_{i}(\mu_{R}), \gamma_{R}, R) = (g_{0M}/\alpha) e^{\gamma_{0M}(\mu_{M})} \]
\[ \sum_{i} \sum_{j_{M} \in S_{M}} \sum_{j_{F} = 0}^{H_{F}} M(i, j_{F}, j_{M}; w_{i}(\mu_{R}), \gamma_{R}, R) = g_{0M} n_{iM} e^{\gamma_{0M}(\mu_{M}) + \gamma_{MR}(\mu_{R})}, \ell = 1, \ldots, L. \]

(29)
for the case with perfectly inelastic demand.

4.3. Alternative concepts of equilibrium

The simple concept of equilibrium adopted and the assumption that any individual has access to the same (and total) set of jobs of a given type are admittedly a methodological simplification. In terms of economic interpretation, they are consistent with a competitive scenario. In Section 2 we have mentioned the possibility of empirically implementing, with appropriate data, more complex and general frameworks such as the one developed in Dagsvik (2000). However, even within the limits of the approach adopted in this paper, many extensions are feasible. For example one might depart from the competitive scenario and assume that the wage rates and the number of available jobs are set by the employers or the unions (collectively acting as a monopolist) or by a bargaining among them. Such alternative assumptions would of course require a suitable reformulation of the equilibrium conditions.

5. AN EMPIRICAL ILLUSTRATION

We illustrate the procedure presented in the previous sections with a simulation of various hypothetical reforms of income support in Italy, using a microeconometric model of household labour supply. The model, the estimates, the policy motivations and the simulated reforms are fully described in Colombino (2011). Here we illustrate the main features of the model and some of the simulation results with the perspective of illustrating the implications of the equilibrium simulation procedure presented in Sections 3 and 4.

5.1. The model

We consider households with two decision-makers (couples) or one decision-maker (singles). The choices of other people – if any – in the household are taken as exogenous.

The choice probabilities for singles and couples are those of expressions (18) and (22) respectively.

Each individual (single or partner in a couple) chooses among 11 job-types defined by weekly hours of work $h$: so $h_0 = 0$ and $h_1, h_2, \ldots, h_{10}$ are ten random values drawn from the intervals 1-8, 9-16, 17-24, 25-32, 33-40, 41-48, 49-56, 57-64, 65-72, 73-80.

For the systematic part of the utility function we adopt a quadratic specification, where $C$ denotes household total net available income and $t$ denotes total available time.
for couples and

\[
V = \theta_C C + \theta_F (t - b_F) + \theta_M (t - b_M) + \theta_{CC} C^2 + \theta_{FF} (t - b_F)^2 + \\
+ \theta_{MM} (t - b_M)^2 + \theta_{CM} C(t - b_F) + \theta_{FM} (t - b_F)(t - b_M)
\]

(30)

for singles (x = F, M).

Some of the above parameters \( \theta \) are made dependent on socio-demographic characteristics (partners’ age, children’s number and age).

Wage rates for those who are observed as not employed are imputed on the basis of a wage equation estimated on the employed subsample and corrected for sample selection.

The data used for the estimation and the simulation exercise were produced starting from a EUROMOD dataset in turn based on the 1998 Survey of Household Income and Wealth (SHIW1998). The EUROMOD Microsimulation model is also used to compute the value of \( C \) for all the job-types.

The data include couples and singles. Both partners of couple households and heads of single households are aged 20 – 55 and are wage employed, self-employed, unemployed or inactive (students and disabled are excluded). As a result we are left with 2955 couples, 366 single females and 291 single males.

The simulation exercise accounts for equilibrium between the total number of available market jobs and the number of people willing to be matched to those jobs. The implicit (simplifying) assumption in the exercise is that, whilst the number of jobs and people willing to work are equated by the equilibrium wage distribution, the hours worked accommodate households’ preferences. For gender \( x = F, M \) we adopt – again for illustrative purposes - the following simple empirical specification for expression (24):

\[
J_x = K_x \mu_x^{-\eta}
\]

(32)

where \( \mu_x \) is the mean of the wage rates distribution for gender \( x = F, M \), \( K_x \) is a gender-specific constant and \( -\eta \) is the elasticity of labour demand. More generally one could allow both for more moments characterizing the wage distribution and for gender-specific direct and cross
elasticities. Expressions (32) and (23) imply:

\[ \gamma_{\tilde{J}}(\mu_x) = \ln\left(\frac{K_x \mu^{-\eta}}{\langle \tilde{g}_{0x} / \alpha \rangle}\right) \quad (33) \]

Given \( J_x \) (observed or simulated under the current tax-transfer system), \( \mu_x \), the estimated \( \gamma_{\tilde{J}} \) and an imputed value of \( \eta \), we can use expressions (32) and (33) to retrieve \( \langle \tilde{g}_{0x} / \alpha \rangle \) and \( K_x \). In this exercise we use four alternative values \( \eta = 0, 0.5, 1, \infty \).

The equilibrium conditions derived in Section 4 are fulfilled by iteratively calibrating \( \mu_x \) (i.e. shifting the location of the wage rate distributions) or directly \( \gamma_{\tilde{J}} \) (when \( \eta = \infty \)) in the course of the simulation.

5.2. The policies

Many analysts have suggested that the current Italian system of income support policies is defective with respect to both efficiency goals (e.g. minimizing distortions and supporting labour mobility) and equity goals (e.g. reducing poverty and economic insecurity). A main deficiency is the lack of a universal income support mechanism.

In this paper we consider various versions of hypothetical income support policies that are universal, i.e. not conditional upon professional or occupational categories or on bargaining or contingent financial constraints. These reforms are stylized cases representative of the different scenarios that are discussed or even actually implemented in many countries. More specifically we focus here upon the issue of comparing means tested transfers vs. non means tested transfers.

In the following description of the policies there appears a threshold \( G \) that is defined as follows. Let us preliminarily define:

\[ C_i = \text{total net available income (current) of household } i; \]

\[ N_i = \text{total number of components of household } i; \]

\[ \tilde{C}_i = C_i / \sqrt{N} = \text{“individual-equivalent” income, i.e. the income imputed to each member of household } i; \]

\[ P = \text{median}(\tilde{C}_i) / 2 = \text{Poverty Line.} \]
Then:

\[ G_i = aP\sqrt{N_i}, \]

where \( a \in [0,1] \) is a “coverage” rate, i.e. the proportion of the “adjusted” poverty line \( P\sqrt{N_i} \) that is covered by \( G_i \). For each reform we simulate three versions with different values of \( a \): 1, 0.75 and 0.50. For example, \( G=0.5P\sqrt{3} \) means that for a household with 3 components the threshold is \( \frac{1}{2} \) of the Poverty Line times the equivalence scale \( \sqrt{3} \).

We consider the following two types of policies.

**Guaranteed Minimum Income (GMI).** Each individual (partner in a couple household or head of a single household) receives a transfer equal to \( G - I \) if single or \( G/2 - I \) if partner in a couple provided \( I < G \) (or \( I < G/2 \)), where \( I \) denotes individual taxable income. This is the standard conditional (or means-tested) income support mechanism.

**Unconditional Basic Income (UBI).** Each individual receives an unconditional transfer equal to \( G \) if single or \( G/2 \) if partner in a couple. It is the basic version of the system discussed for example by Van Parijs (1995) and also known in the policy debate as “citizen income” or “social dividend” (Meade 1995; Van Trier 1995).8

The income support mechanisms are coupled with a progressive tax that replicates a simplified version of the current system where the labour income marginal tax rates are applied to the whole income exceeding \( G \) (or \( G/2 \)) and proportionally adjusted according to a constant \( \tau \) in order to fulfil the public budget constraint)9. Altogether we have 2 (types) \times 3 (values of \( a \)) = 6 reforms.

Each reform defines a new budget constraint for each household. The simulation consists of running the model after replacing the current budget constraint with the reformed one. The parameter \( \tau \) (defined above) is endogenously determined so that the total net tax revenue is equal to the one collected under the current tax-transfer system (taking into account the households’ behavioural responses). The equilibrium conditions are attained by iteratively calibrating the mean of the wage rate distribution: this process determines the number of available market jobs through expression (32) and the value of \( \mu \) (expression (33)), which in turn affect the number of people matched to a market job according to expressions (18) and (22). Five simulation procedures are adopted: one where the equilibrium conditions are ignored (No-Equilibrium) and
four where the equilibrium conditions are alternatively determined by $\eta = 0, 0.5, 1.0, \infty$.

Besides the 6 alternative reforms we also simulate a tax-benefit system \textit{(Pre-reform)} with the same five alternative equilibrium procedures used for the reforms: the \textit{Pre-reform} system it is characterized by the same income support mechanism as in the true current system, but the tax rule is the simplified version also adopted for the reforms. Therefore we compare what would happen with this system and with the reforms under the alternative equilibrium conditions. We think this procedure provides a comparison between reforms that is more consistent than the standard method consisting of comparing the observed status quo to the reforms\textsuperscript{10}.

We rank the policies with the Gini Social Welfare Function. In order to define this function we first recall the concept of Expected Maximum Utility (EMU): it is the expectation of the maximum utility attained under a given tax-transfer regime. If we consider for simplicity the model of Section 3, it can be shown that the EMU of household $i$ in the equilibrium induced by regime $R$ is equal to \[\ln \left( \sum_{h=0}^{H} \exp \left\{ V(i, h; \mu_R), R + \gamma_R(\mu_R)I(\hat{k} > 0) \right\} \right),\] i.e. the natural logarithm of the denominator of the matching probabilities\textsuperscript{11}. Next, the Individual Welfare of household $i$ is defined as the money metric equivalent of the EMU, i.e. the level of income that makes the EMU of the reference household (we choose the worst-off one) equal to the EMU of household $i$ (King 1983, Colombino 2011). Last, the Gini Social Welfare Function is defined as \((\text{Average Individual Welfare}) \times (1 - \text{Gini index of the distribution of Individual Welfare})\). This is similar to the so-called Sen Social Welfare index (which however is defined on income rather than on welfare) and it can be rationalized as a member of the class of rank-dependent social welfare indexes.\textsuperscript{12}

5.3. Results

Table 1 reports some results of the simulations. The reforms are identified by the acronym of the income support mechanism (GMI and UBI) and by the coverage, i.e. percentage value of $a$ (50, 75 or 100) defined in section 5.2. For example, UBI-75 denotes a policy where the income support mechanism is UBI and the transfer $G$ is 75\% of the Poverty line.

For each reform we report three pieces of information related to behavioural effects (average net household income), costs and distortions (top marginal tax rate) and distributive effects (poverty ratio) of the reforms.

Although the main purpose of the empirical exercise reported in this paper is the illustration of
the equilibrium simulation procedure rather than the specific evaluation of reforms, some policy-relevant results are worthwhile a comment. The current mechanism of income support is always ranked at the bottom, except when $\eta = \infty$. The most realistic scenarios ($\eta = 0.5$ or $1.0$) suggest UBI-50 as the best reform. UBI-50 would bring down the poverty ratio from about 4.4% to about 0.5% and would require a top marginal tax rate of about 51%.

From the methodological point of view – the focus of this paper – there are many aspects of the results that reveal the potential relevance of the equilibrium simulation.

A first important methodological perspective emerges from comparing the no-equilibrium and ($\eta = \infty$) simulation. The point concerns the interpretation of the simulations from the point-of-view of comparative statics. Especially when simulating institutions or reforms (i.e. not temporary changes) with cross-section data, the most appealing interpretation is that we are comparing different equilibria, i.e. configurations of agents’ choices that are somehow mutually consistent. As we have noted in Section 3, the common practice of performing behavioural simulation while leaving the wage rates unchanged might seem – incorrectly – interpretable as consistent with a perfectly elastic demand scenario. Table 1 confirms that this interpretation in general is not appropriate – as explained in Section 3: the simulation performed under the correctly specified scenario with perfectly elastic demand produces a ranking of policies that is very different from the one produced by the no-equilibrium simulation.

A second useful perspective is comparing the perfectly inelastic demand case ($\eta = 0$), the elastic demand cases ($\eta = 0.5, 1.0$) and the perfectly elastic demand case ($\eta = \infty$).

1. If we look at the Gini Social Welfare ranking, we see that the perfectly inelastic scenario leads to choosing a generous means-tested mechanism (GMI-100) as the best reform, while the elastic and perfectly elastic scenarios favours a less generous unconditional mechanism (UBI-50). In general, scenarios with more elastic labour demand seem to favour UBI mechanisms over GMI mechanisms. This is so because a more elastic demand allows less constrained choices and more variability in the number of jobs (and workers). As an implication, the perverse effects of the poverty trap on labour supply and income, present with GMI but not with UBI, have more space to manifest themselves. Also the unconditional transfers of UBI have a negative effect on labour supply and income but they are much more effective than GMI in reducing the Poverty Ratio.

2. A less elastic demand requires higher equilibrium wages, which in turn lead to higher net household incomes. The perfectly elastic demand case instead implies unchanged wage
rates and lower net incomes, also due to some reduction in labour supply.

3. With increasing $\eta$, less generous policies – including the current one – move up in the ranking. This happens because a more elastic labour demand moderates the increase in equilibrium wages, therefore implying higher equilibrium marginal tax rates. Also, when $\eta$ approaches $\infty$, the alternative between conditional or non-conditional transfers seems to become less important than the generosity of the transfer.
### Table 1  Effects of the reforms according to the different simulation procedures

<table>
<thead>
<tr>
<th></th>
<th>Rank based on the Gini Social Welfare Function</th>
<th>Monthly Average Household Net Income</th>
<th>Top Marginal Tax Rate (%)</th>
<th>Head-Count Poverty Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N. E.</td>
<td>$\eta=0$</td>
<td>$\eta=0.5$</td>
<td>$\eta=1$</td>
</tr>
<tr>
<td>Pre-Reform</td>
<td>7°</td>
<td>7°</td>
<td>7°</td>
<td>7°</td>
</tr>
<tr>
<td>GMI-50</td>
<td>4°</td>
<td>4°</td>
<td>4°</td>
<td>4°</td>
</tr>
<tr>
<td>GMI-75</td>
<td>6°</td>
<td>2°</td>
<td>5°</td>
<td>5°</td>
</tr>
<tr>
<td>GMI-100</td>
<td>5°</td>
<td>1°</td>
<td>6°</td>
<td>6°</td>
</tr>
<tr>
<td>UBI-50</td>
<td>2°</td>
<td>3°</td>
<td>1°</td>
<td>1°</td>
</tr>
<tr>
<td>UBI-75</td>
<td>1°</td>
<td>5°</td>
<td>2°</td>
<td>2°</td>
</tr>
<tr>
<td>UBI-100</td>
<td>3°</td>
<td>6°</td>
<td>3°</td>
<td>3°</td>
</tr>
</tbody>
</table>

N.E. = No-equilibrium simulation procedure;  
Head-Count Poverty Ratio = percentage of households below the Poverty Level (as defined in Section 5.2);  
Net Income is measured in Euros.
6. CONCLUSIONS

The standard simulation procedure adopted when using microeconometric models of labour supply for the evaluation of reforms might not be consistent with an interpretation of the simulation results in terms of comparative statics, i.e. comparison of different equilibria. This happens when the model includes a representation of aspects of the pre-reform equilibrium (such as the availability of different types of jobs) that are going to change in the post-reform equilibrium but these changes are not properly accounted for. We have proposed a simulation procedure that takes into account such changes and leads to a consistent interpretation of the simulation results as an exercise in comparative statics. We have adopted a very simple notion of equilibrium, but the same procedure can accommodate different and more complex ones. We have illustrated the relevance of the different simulation procedures with an evaluation of alternative reforms of the Italian income support policies. The results show large differences in the ranking of policies and in the behavioural and fiscal effects, especially when comparing the two opposite cases of perfectly inelastic and perfectly elastic labour demand.

ACKNOWLEDGEMENT

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The empirical example of this paper uses EUROMOD (Ver. 27a). EUROMOD is a tax-benefit microsimulation model for the European Union that enables researchers and policy analysts to calculate, in a comparable manner, the effects of taxes and benefits on household incomes and work incentives for the population of each country and for the EU as a whole. EUROMOD was originally designed by a research team under the direction of Holly Sutherland at the Department of Economics in Cambridge, UK. It is now developed and updated at the Microsimulation Unit at ISER (University of Essex, UK).
REFERENCES


COLOMBINO A new equilibrium simulation procedure with discrete choice models

1 The practice is common in many area of application of discrete choice models (transportation, child-care, education etc.) where different alternatives may have different accessibility.

2 The derivation of the Conditional Logit expression for utility maximization under the assumption that the utility random components are i.i.d. Type I extreme value is due to McFadden (1974). The first applications of models belonging to the Conditional Logit family to labour supply choices are due to Aaberge et al. (1995) and Van Soest (1995).

3 A different procedure for equilibrium simulation – which however would not be appropriate for the class of microeconometric models with “dummies refinement” considered here – has been proposed by Creedy and Duncan (2005).


5 More recent datasets are of course available. We chose to use a model that was already estimated on 1998 data with the main purpose of illustrating a methodological proposal.

6 See for example Onofri (1997), Baldini et al. (2002), Boeri and Perotti (2002) and Sacchi (2005). A recent empirical analysis of the distortions that characterize the Italian income support policies and of some proposed reforms is provided by Colonna and Marcassa (2012). A first microeconometric evaluation of alternative reforms of the Italian tax-transfer system was done by Aaberge et al. (2004). In 2012 the Italian Parliament has approved a reform of the income support policies that contains some steps toward universalism, although so far it does not change the basic characteristics of the current system.

7 The “square root scale” is one of the equivalence scales commonly used in OECD
publications.

Colombino (2011) extensively discusses positive and negative implications of GMI and UBI as well as of other possible reforms.

In the true current system some incomes (e.g. capital income) are taxed according to a different rule.

The results reported in Colombino (2011) are in part different from the ones reported here since the current system is defined there as the observed status quo.

The proof and the illustration of this result can be found in many sources, e.g. Ben-Akiva and Lerman (1985) or McFadden (1978).


A related, but different, issue concerns the time need to reach a new equilibrium, which cannot be addressed with cross-section data.